Numerical Strip-Yield Calculation of CTOD

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Outline

- CTOD background
- Using Boundary Elements to calculate crack face displacements
  - Theory
  - Practical procedure
  - Example cases
- Summary and future plans

CTOD background: plastic zone sizes

- Irwin (1958)
  - LEFM gives $\sigma \sim 1/\sqrt{r}$; however: real materials yield
  - Crack behaves as if it were longer: $a_{eff}=a+\rho$
  - Plastic zone size estimated from stress redistribution

- Dugdale (1960)
  - Yielding confined to narrow strip ahead of crack (the “strip yield” model)
  - Stresses at “effective” crack tip ($a+\rho$) are finite
    - ⇨ Yield zone loading neutralizes stress singularity due to remote loading
    - ⇨ plastic zone size estimated from setting $K(a+\rho)=0$

CTOD background: plastic zone sizes, cont'd

- Knowledge of ρ enabled derivation of explicit CTOD expression
  - Complex-variable analysis used (no full elastic-plastic analysis)
  - Elastic-plastic behavior modeled by superposition of 2 ***elastic*** solutions

- Wells (1963)
  - CTOD is proportional to overall tensile strain, even after general yielding
    - ⇨ CTOD became widely accepted as a useful fracture criterion when effects of the crack tip plastic zone are important

CTOD background: some calculation methods

- Dugdale’s model
  - Based on thin infinite plate, plane stress, remote tension
  - Extensions to other infinite geometries limited to a few particular cases
  - Arbitrary finite geometries require tailor-made elastic solutions

- Weight function, green’s function, collocation methods
  - Developed for particular finite geometries
  - Potentially heavy computational burden (e.g. reference solutions)

- Finite elements
  - General-purpose, but also severe computational toll
  - Where behind the crack tip to measure CTOD?
    - ⇨ 1st node, 2nd node, 45° intercept, or some prescribed distance
  - Re-meshing burden for analyses of multiple loads or cracks

Using Boundary Elements to calculate crack face displacements: theory

- Direct application of conventional BEMs to fracture problems leads to mathematically degenerate formulation
  - Cause: geometric proximity of crack surfaces
  - Information about crack face tractions is lost
  - Can circumvent by developing additional integral equation for crack face tractions

- One approach:
  - Derive crack face traction equation from displacement eq^n via
    - ⇨ Strain-displacement relations, Hooke’s law, limiting process
  - Resulting equation contains hypersingular kernel
    - ⇨ Requires special interpretation; challenging to evaluate numerically

Using Boundary Elements to calculate crack face displacements: theory (cont'd)

- Better approach (Prof. Mear et al, Univ of Texas):
  - Hypersingularity avoided by eliminating the offending terms in the displacement equation **before** deriving traction equation:
    - ⇨ Appropriate choice of stress function for the stress kernel
    - ⇨ Integration by parts to obtain a "modified" displacement equation
  - Crack face traction equation is then derived as before (strain-displacement relations, Hooke's Law, limiting process)
  - Other practical benefits: small mesh size and fast solution times
  - This is the basis of NASGRO's BE component

Using Boundary Elements to calculate crack face displacements: theory (cont'd)

- Gradients of relative crack face displacements ΔD
  - Described by dislocation density function A
  - A is approximated by functions containing the requisite singularity
  - A_j are nodal quantities in the vector of unknowns solved by NASBEM

$$A(\zeta) = \frac{i\mu}{\pi(\kappa+1)} \frac{\partial[\Delta D(\zeta)]}{\partial s}$$

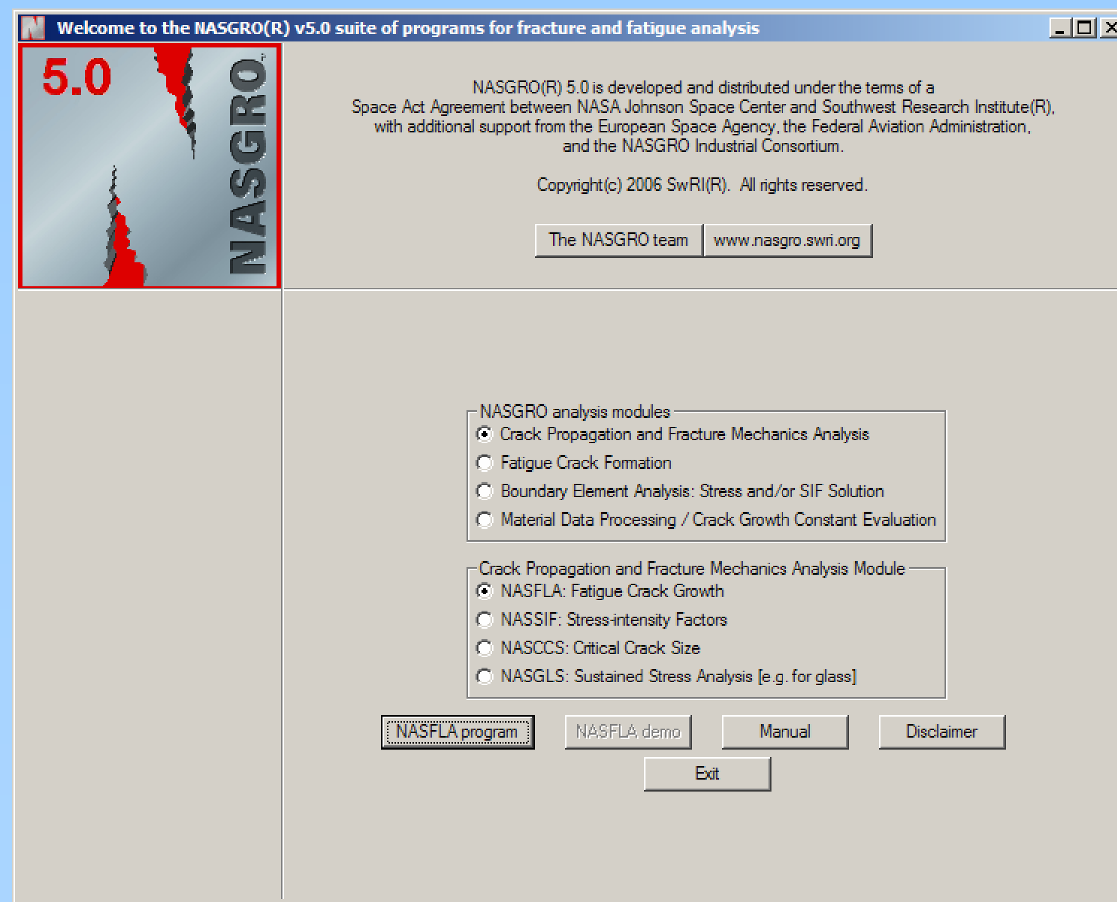
$$A(t) = A[\zeta(t)] = \frac{1}{2\sqrt{\alpha_j}}\left(\frac{1-t}{\sqrt{\rho_j+t}}A_j + \frac{1+t}{\sqrt{\rho_j+t}}A_{j+1}\right)$$

- Technique implemented in NASBEM to integrate A
  - ΔD is sum of contributions from each crack element between the tip and the point of interest

$$\Delta D(\zeta) = \frac{\pi(\kappa+1)}{i\mu} \int_0^s A[\zeta(s)]ds$$

NASBEM is NASGRO's Boundary Element Analysis module

- NASGRO is an analysis software suite with four distinct modules:
  - Fracture mechanics and fatigue crack growth analysis (NASFLA series)
  - Fracture and fatigue crack growth material property database; fitting of experimental data (NASMAT)
  - 2D boundary element stress analysis and stress intensity factor calculation (NASBEM)
  - Fatigue crack formation (initiation) analysis (NASFORM)

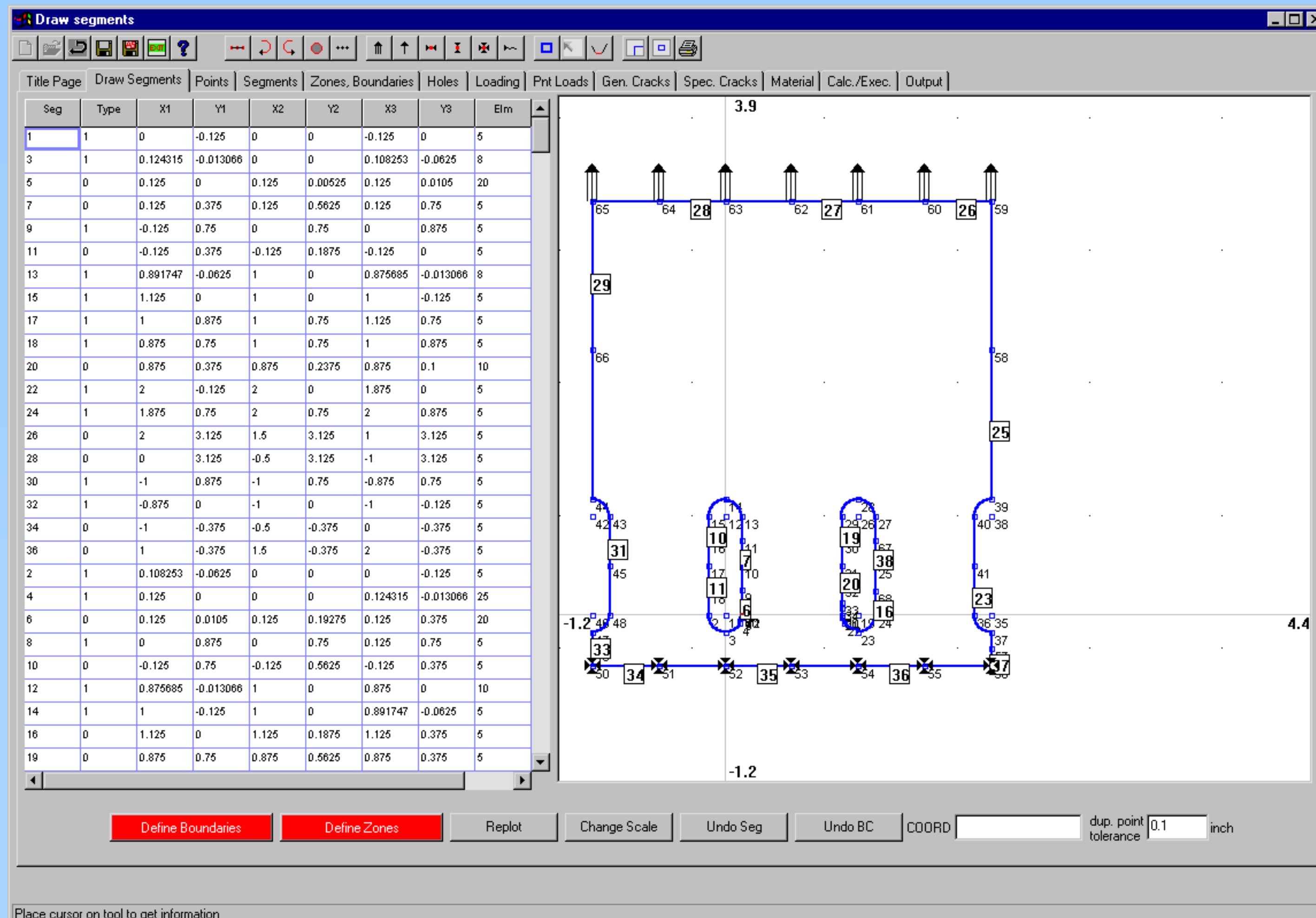


NASGRO history

- 1980s:
  - NASA/FLAGRO development initiated to provide fracture control analysis for manned space programs
  - NASA Fracture Control Methodology Panel formed to standardize methods and monitor NASA/FLAGRO development
- 1990s:
  - NASA Interagency Working Group (NASA, DoD, FAA, ESA) formed to provide guidance for NASA/FLAGRO development
  - Additional NASA, FAA, USAF support for aging aircraft
- 2000s:
  - NASA and Southwest Research Institute® sign Space Act Agreement for joint NASGRO development
  - NASGRO Industrial Consortium formed by SwRI; members include government agencies and industrial representatives

Example of typical NASBEM use: Orbiter feedline flowliner

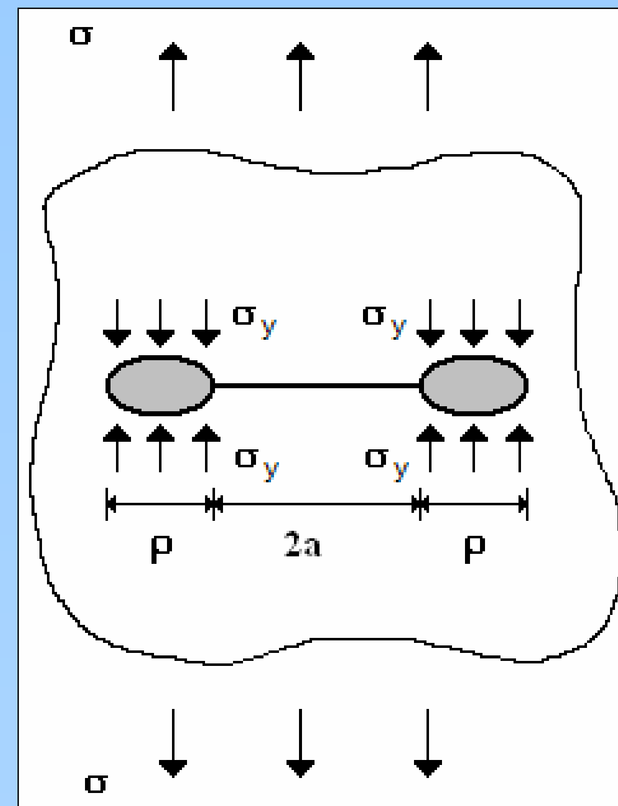
- Fatigue cracks in flowliner (LH_2 supply to SSME)
  - 1’ Ø, 8-12’ L
  - Bellows within gimballing joints
  - Flowliners inside bellows to smooth flow
- Concern:
  - Engine failure due to debris
  - Loss of mission or vehicle
- NASBEM used to get *K* vs *a*



Seg	Type	X1	Y1	X2	Y2	X3	Y3	Elm
1	1	0	-0.125	0	0	-0.125	0	5
3	1	0.124315	-0.013066	0	0	0.108253	-0.0625	8
5	0	0.125	0	0.125	0.00525	0.125	0.0105	20
7	0	0.125	0.375	0.125	0.5625	0.125	0.75	5
9	1	-0.125	0.75	0	0.75	0	0.875	5
11	0	-0.125	0.375	-0.125	0.1875	-0.125	0	5
13	1	0.891747	-0.0625	1	0	0.875685	-0.013066	8
15	1	1.125	0	1	0	1	-0.125	5
17	1	1	0.875	1	0.75	1.125	0.75	5
18	1	0.875	0.75	1	0.75	1	0.875	5
20	0	0.875	0.375	0.875	0.2375	0.875	0.1	10
22	1	2	-0.125	2	0	1.875	0	5
24	1	1.875	0.75	2	0.75	2	0.875	5
26	0	2	3.125	1.5	3.125	1	3.125	5
28	0	0	3.125	-0.5	3.125	-1	3.125	5
30	1	-1	0.875	-1	0.75	-0.875	0.75	5
32	1	-0.875	0	-1	0	-1	-0.125	5
34	0	-1	-0.375	-0.5	-0.375	0	-0.375	5
36	0	1	-0.375	1.5	-0.375	2	-0.375	5
2	1	0.108253	-0.0625	0	0	0	-0.125	5
4	1	0.125	0	0	0	0.124315	-0.013066	25
6	0	0.125	0.0105	0.125	0.19275	0.125	0.375	20
8	1	0	0.875	0	0.75	0.125	0.75	5
10	0	-0.125	0.75	-0.125	0.5625	-0.125	0.375	5
12	1	0.875685	-0.013066	1	0	0.875	0	10
14	1	1	-0.125	1	0	0.891747	-0.0625	5
16	0	1.125	0	1.125	0.1875	1.125	0.375	5
19	0	0.875	0.75	0.875	0.5625	0.875	0.375	5

Using NASBEM to calculate CTOD: procedure

- Use NASBEM to construct model
  - “Mathematical” crack consisting of
    - ⇨ Physical crack *a*
    - ⇨ Cohesive load zone *ρ*
  - Applied loading
  - Cohesive yield loading

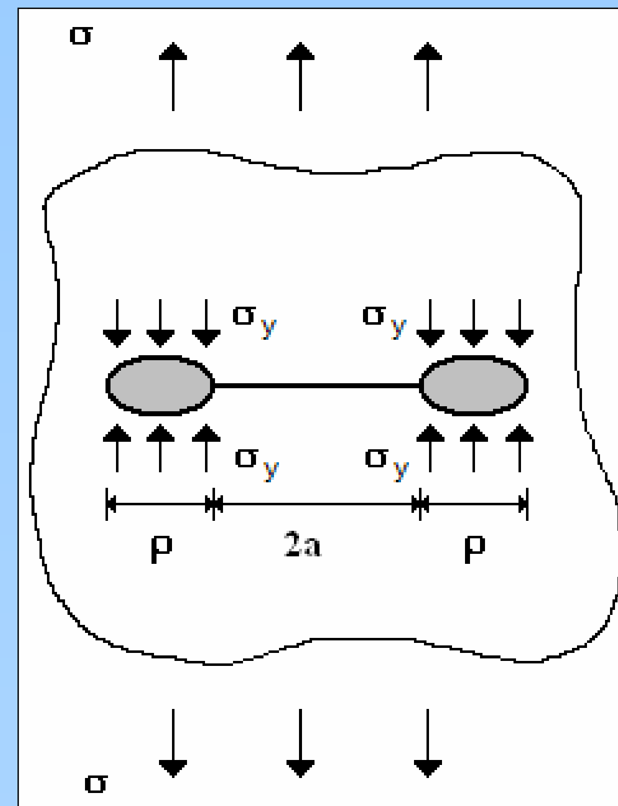


- Following Dugdale’s idea
  - Plastic zone is sized so that K due to cohesive loading cancels K due to applied loading:
    $K_{\sigma y} = -K_{\sigma}$

Using NASBEM to calculate CTOD: procedure (cont'd)

- For a given yield stress σ_Y, achieve $K(a+\rho) = K_{\sigma y} + K_\sigma = 0$
  - by setting the plastic zone size ρ and iterating on the remote stress σ
    - ⇨ advantage: no need to remesh while iterating
  - or by setting the remote stress σ and iterating on ρ
    - ⇨ advantage: CTOD obtained for specific values of σ

- CTOD value is given by crack face displacement at tip of physical crack a

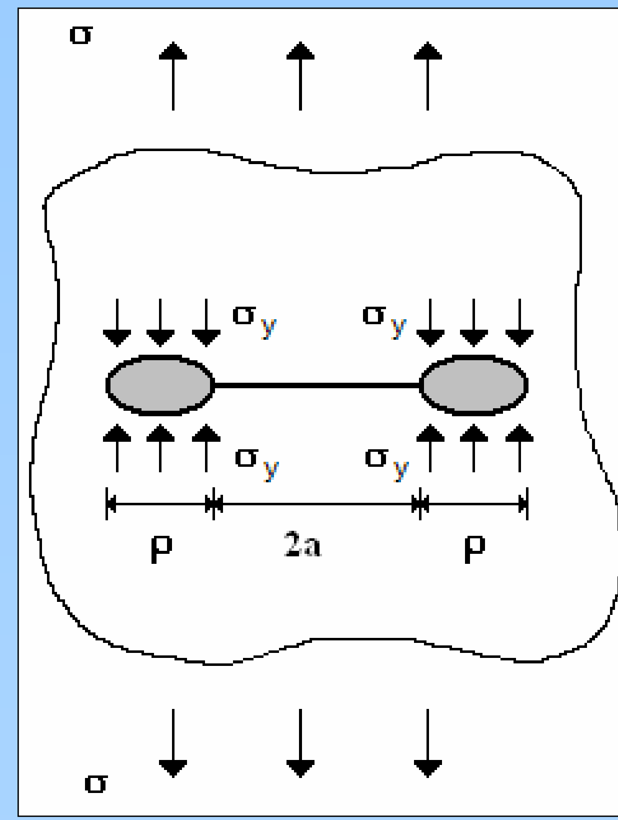


Using NASBEM to calculate CTOD: results

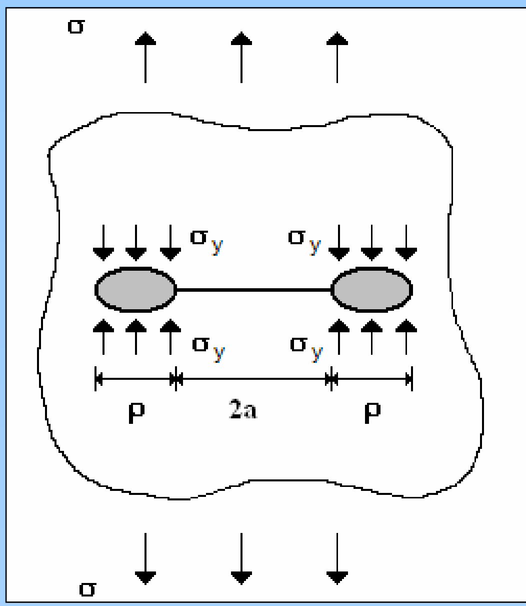
- Mesh
  - Quadratic boundary elements, linear crack elements
- Smaller mesh size than other BEM formulations
  - Typical error <3% with 20 elements or less per boundary or crack
  - Crack face loading discontinuity requires a finer mesh
  - Fast results (example cases run in 2-3 seconds)
- Configurations studied
  - Center crack in finite and infinite sheets
  - Edge crack in finite sheet
  - Cracks from holes in infinite sheets
  - Periodic cracks in infinite sheet
  - 3-hole tension specimen

CTOD verification case: reproducing Dugdale's result

- First verification case
  - Reproducing Dugdale's model really should work!
- Dugdale model consists of
  - Center crack in infinite plate
  - Remote uniform tension
  - Cohesive yield stress on crack faces near crack tips



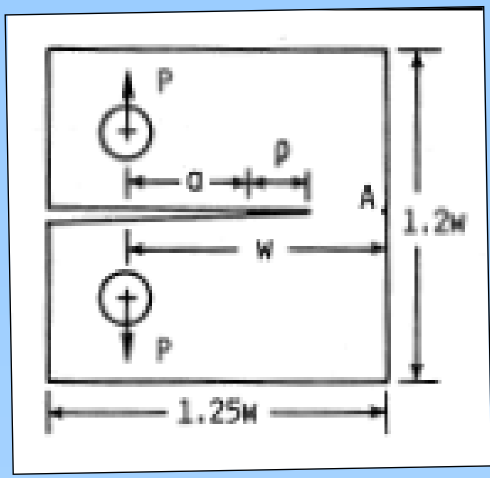
CTOD verification case: reproducing Dugdale's result, cont'd

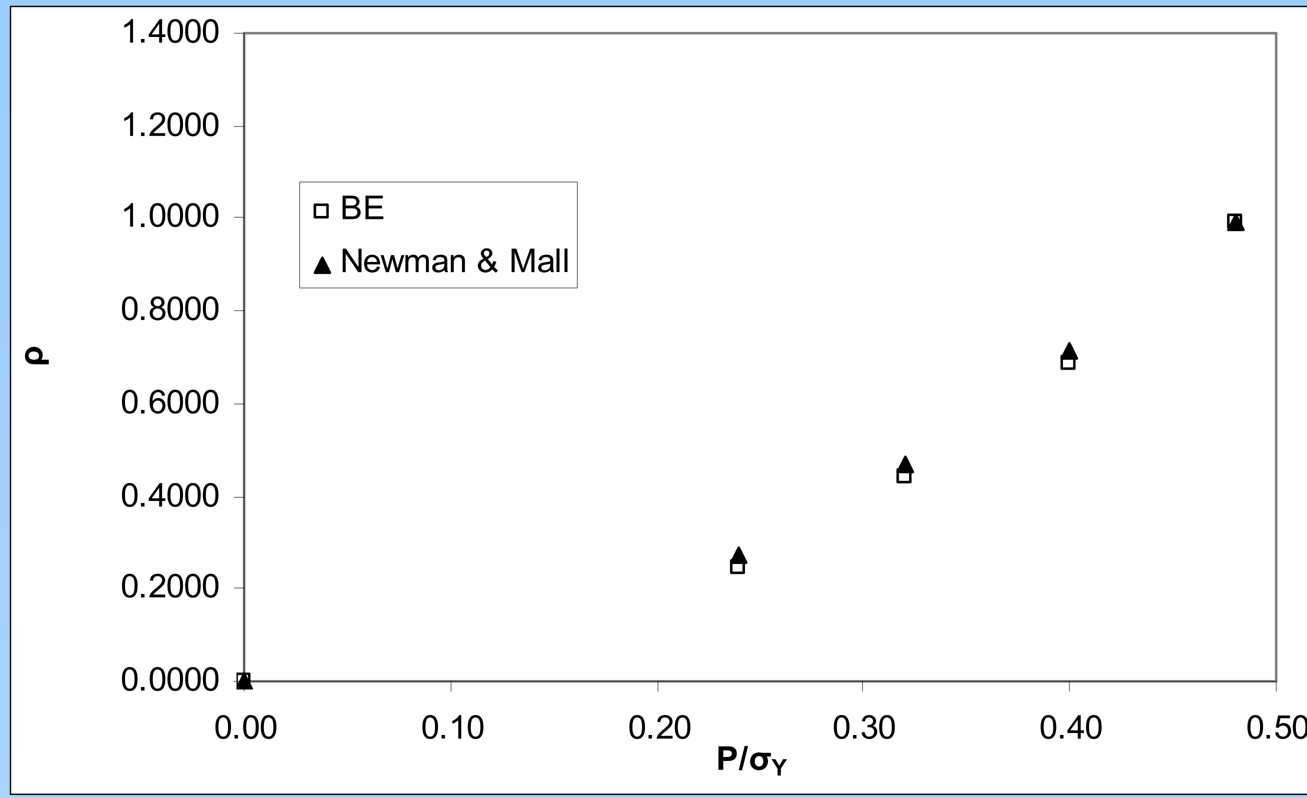


- Results virtually identical over wide range of σ/σ_Y
  - 4 significant digits
  - non-uniform error due to manually iterating to *K=0*

σ / σ_Y	CTOD/($\sigma_Y a/E$) BEM	CTOD/($\sigma_Y a/E$) Dugdale	Error (%)
0.16	0.0810	0.0813	0.37
0.24	0.1845	0.1854	0.49
0.32	0.3362	0.3362	0
0.40	0.5397	0.5397	0
0.48	0.8100	0.8050	0.62
0.56	1.1469	1.1470	0.01
0.64	1.5890	1.5890	0
0.72	2.1740	2.1740	0
0.80	2.9900	2.9900	0
0.88	4.2650	4.2640	0.02
0.96	7.0620	7.0490	0.18

CTOD verification case: edge crack

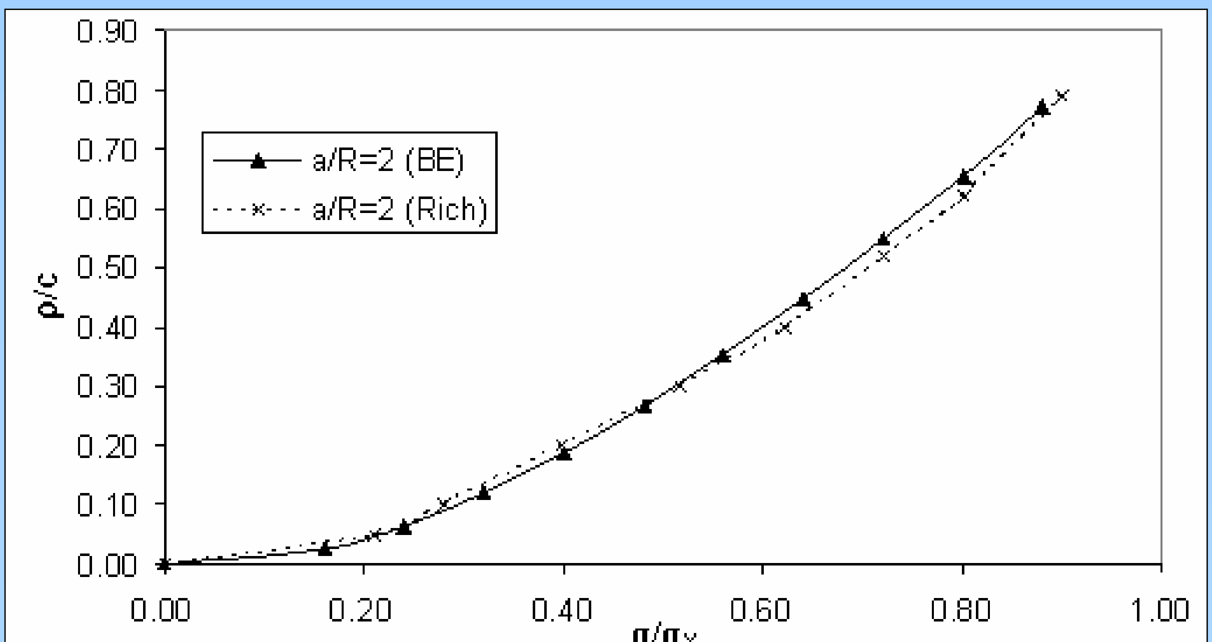


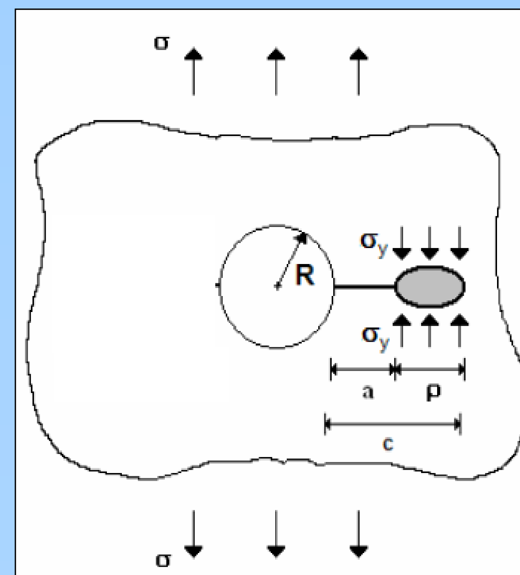


- Verification case: C(T) specimen with W=3, a=1
  - Plastic zone size and CTOD were calculated (ρ is shown here)
  - Excellent agreement with collocation results by Newman & Mall, and also Terada (both 1983)

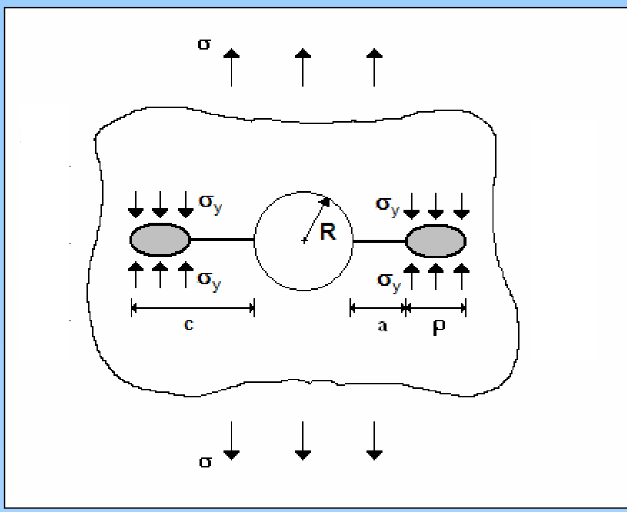
CTOD verification case: 1 crack from a hole, infinite plate

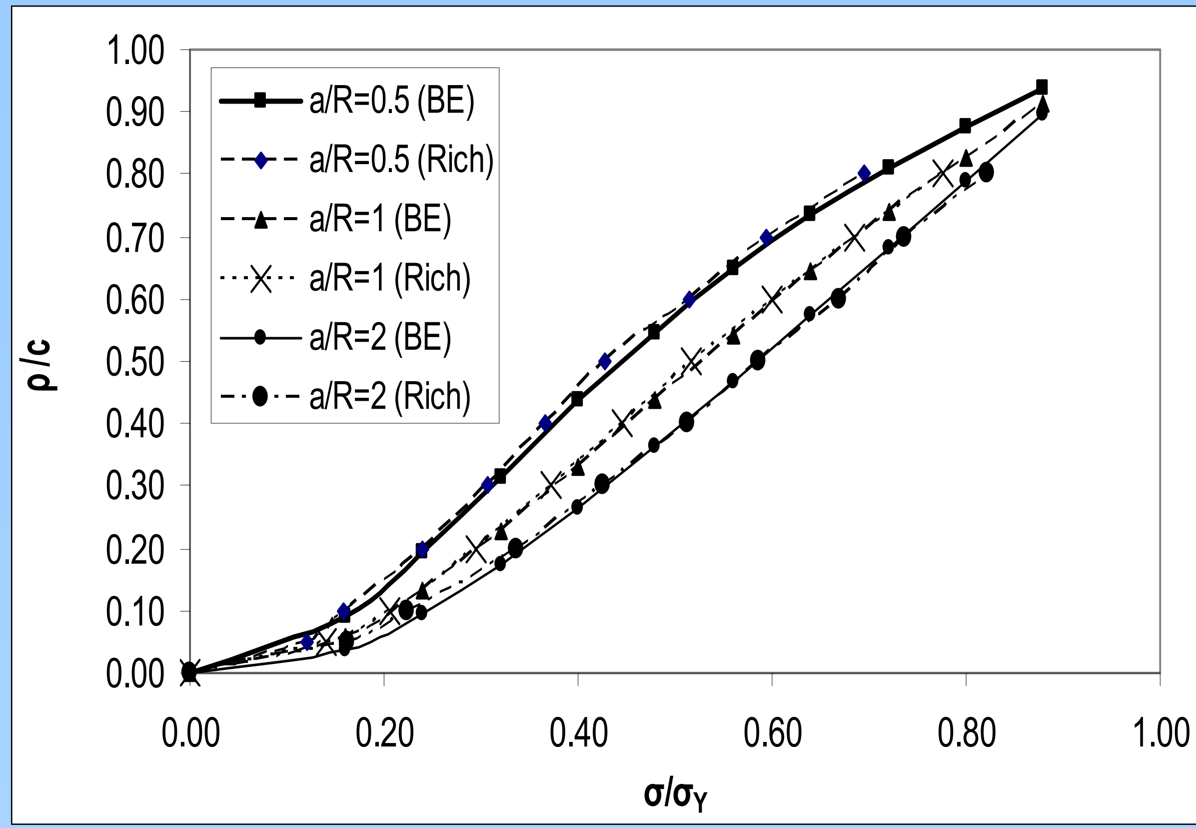
- Verification case: 1 crack from a hole in an infinite plate under remote uniaxial loading
  - Plastic zone size calculated
  - Excellent correlation to analytical results by Rich (complex-variable analysis with conformal mapping, 1968)





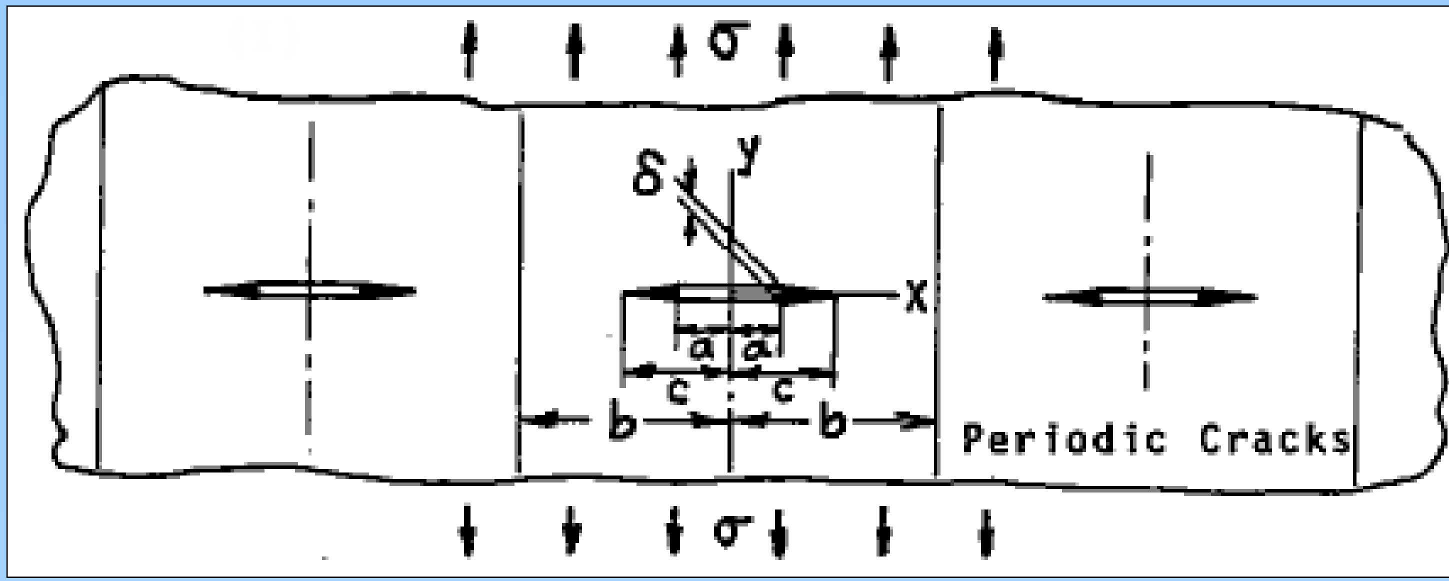
CTOD verification case:
2 cracks from a hole, infinite plate





- Verification case: plastic zone size studied for various values of *a/R*
  - Difference between NASBEM and Rich < 2.5%
  - Larger *ρ* in small cracks due to higher stress concentration at hole
  - Solution approaches Dugdale solution for large *a/R*

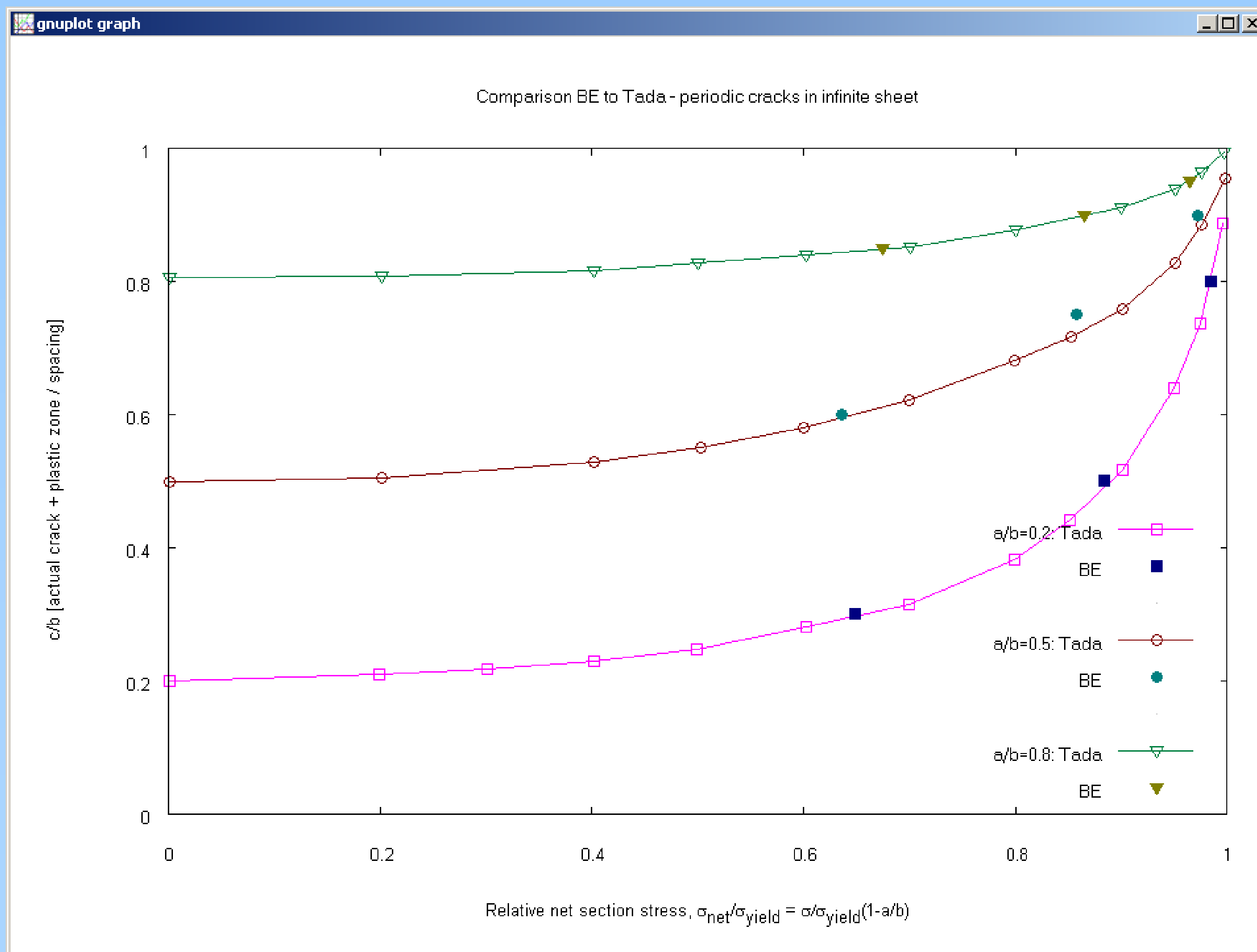
CTOD verification case: periodic cracks in an infinite sheet



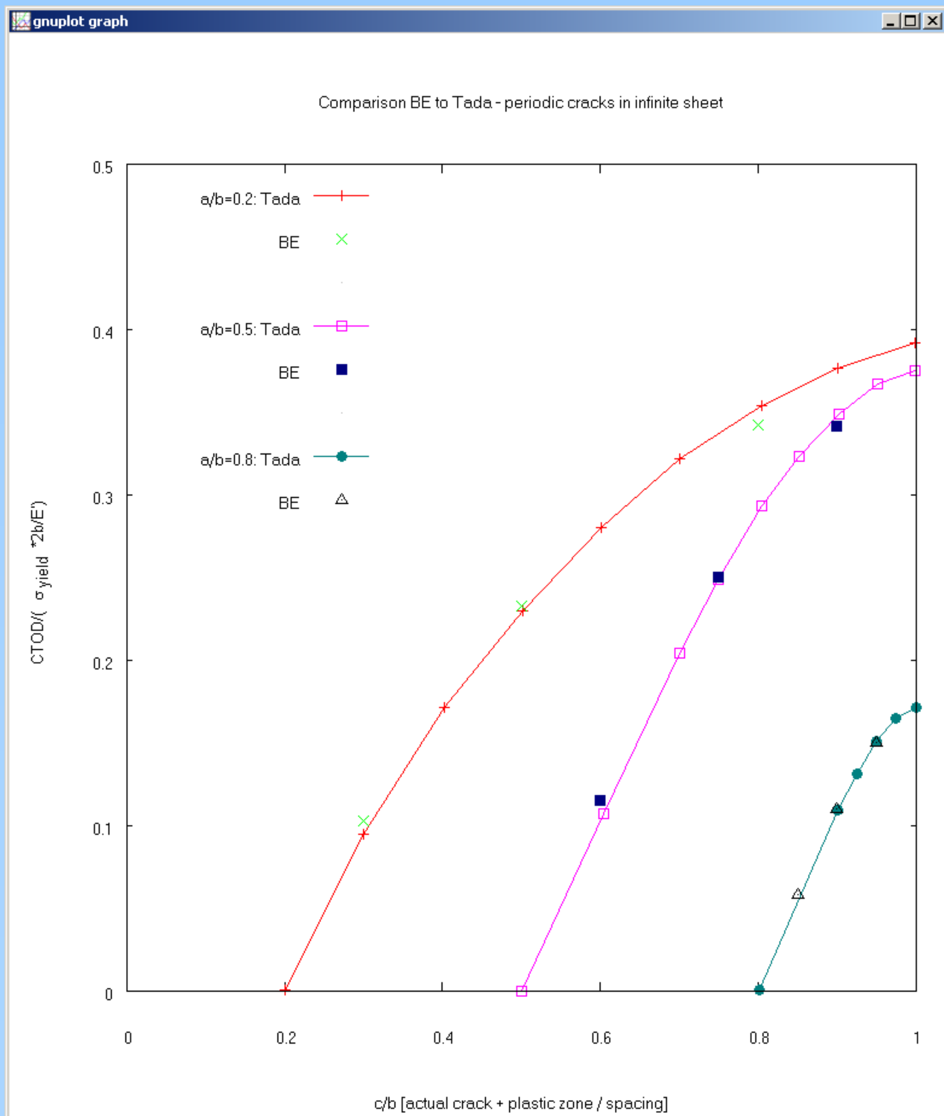
- Practical considerations:
  - How to model an infinite number of cracks?
  - 7 cracks seems a good approximation -- idea taken from literature on modelling large arrays of fuselage fasteners
- BE compared to Tada (Westergaard stress function, 1974)

CTOD verification case: periodic cracks in an infinite sheet, cont'd

- Excellent correlation for plastic zone size v applied stress



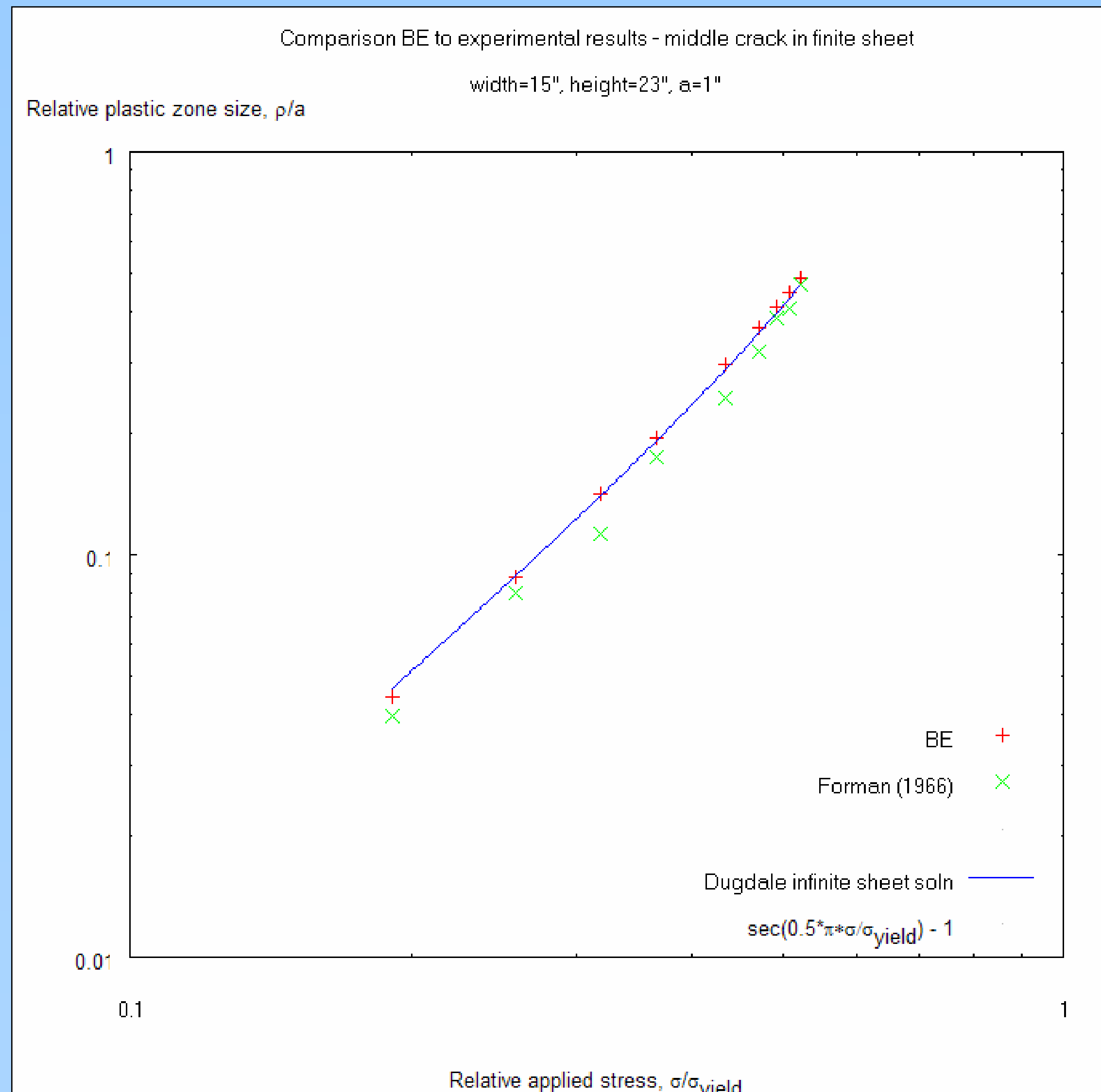
CTOD verification case: periodic cracks in an infinite sheet, cont'd



- Excellent correlation for CTOD v plastic zone size

CTOD calculations: center crack in finite-width sheet

- BE compared to test data (Forman, 1966)
  - Tests on 0.020” AM350CRT steel sheet for toughness variation with specimen size
  - Plastic zone sizes were measured photographically
  - NASBEM compares well with test data



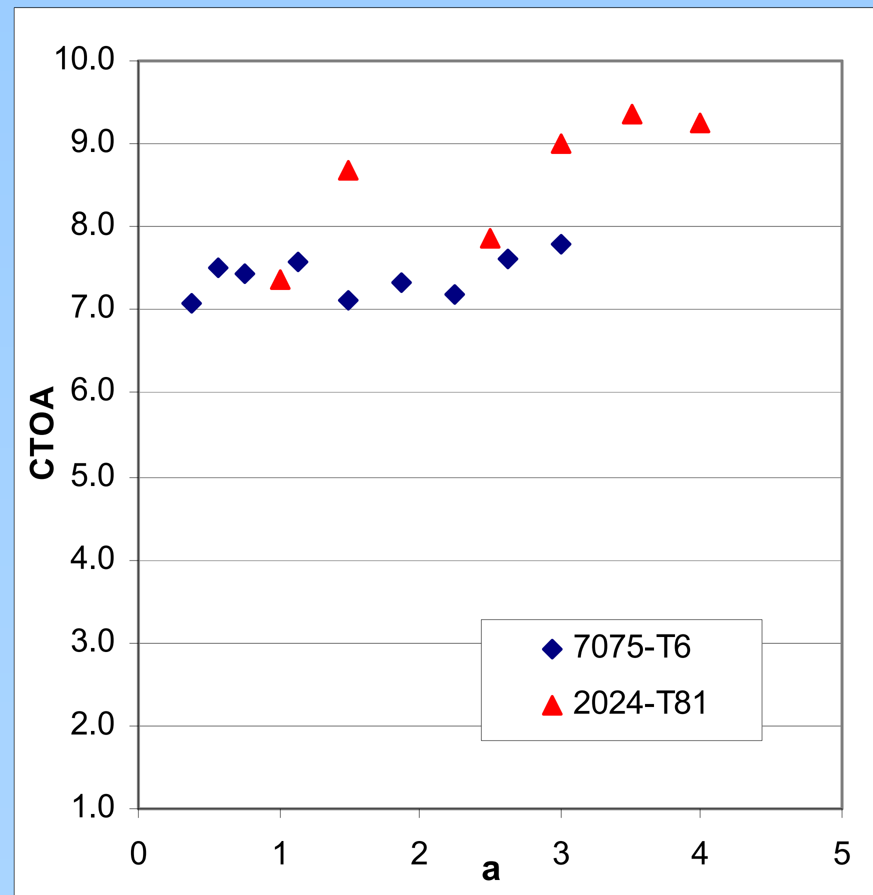
CTOA calculations

- Using NASBEM as a fracture predictor:
  - Crack tip opening angle (CTOA) has been noted by many to be a useful fracture criterion
  - CTOA is calculated at ~0.04” (1 mm) behind the crack tip
    - ⇨ Comparisons to analytical results on previous pages were for CTOD at crack tip (“δ_5”) – not a practical location for real measurements
  - CTOA = 2 * $\tan^{-1}$(CTOD/2x), where x is distance behind crack tip

- Comparisons for
  - M(T) specimen: Al 7075-T6, Al 2024-T81
  - 3-hole tension specimen: Al 7075

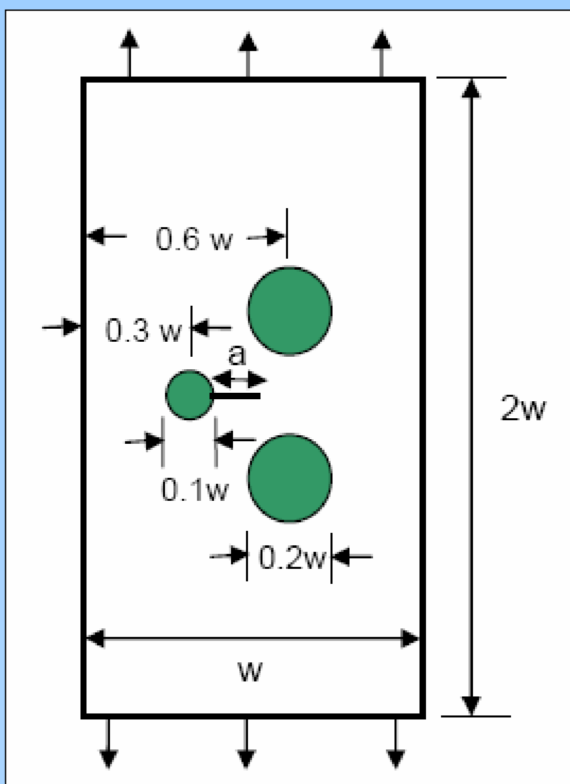
CTOA calculations: center crack in finite-width sheet

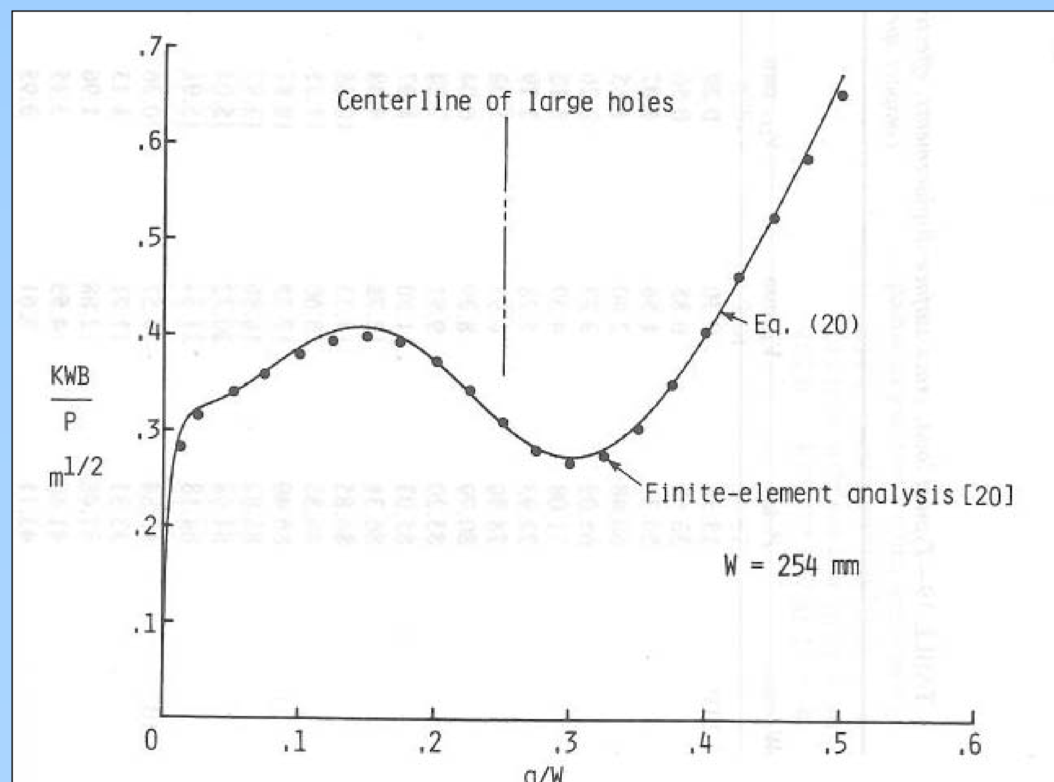
- BE compared to test data (Forman, 1966)
  - M(T) specimens, 0.060” sheet
  - Al 7075-T6, 2024-T81
- Idea was to see if calculated CTOD or CTOA was reasonably constant over crack size *a*
  - looks good for 7075
  - less so for 2024



CTOA calculations: 3-hole tension specimen

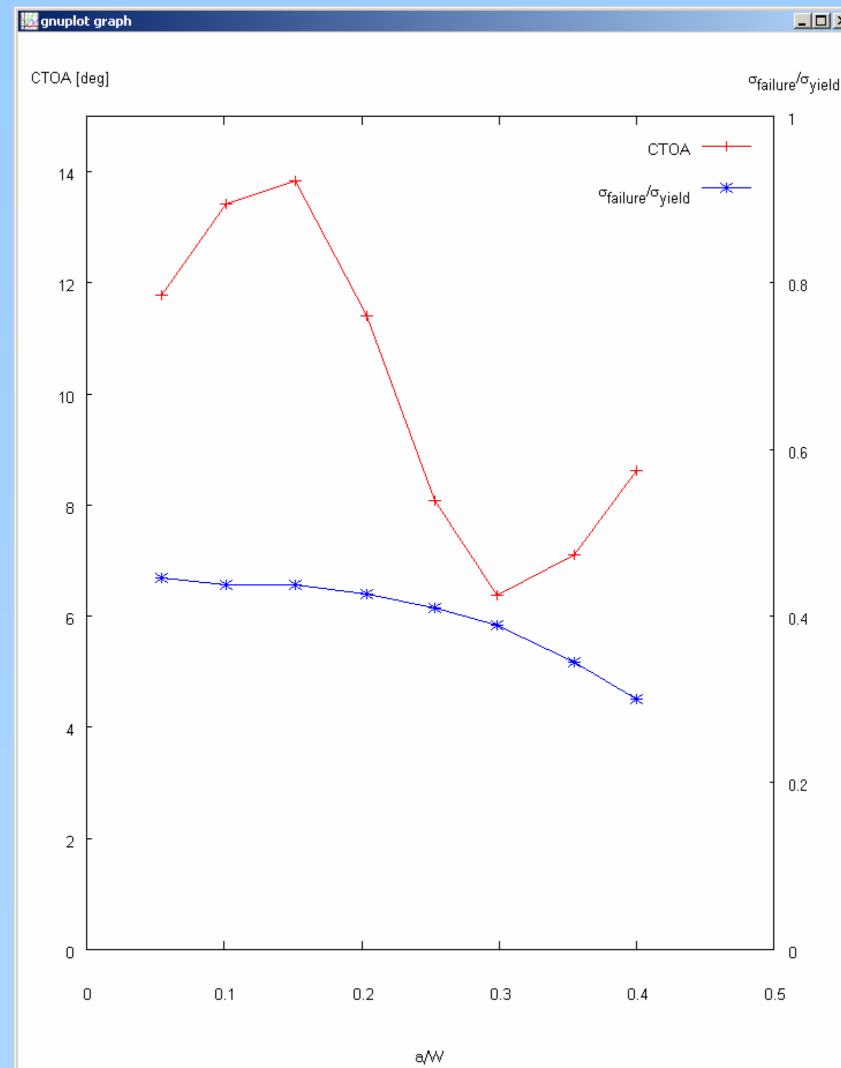
- 3-hole tension (THT) specimen simulates *K* for a cracked stiffened panel
  - *K* curve taken from ASTM STP 896 (1985)





CTOA calculations:
3-hole tension specimen

- CTOA calculated for failure loads taken from ASTM round-robin on experimental and predictive fracture analysis methods (ASTM STP 896)
  - K and calculated CTOA show that THT is a complex configuration
  - Work is in progress



Summary

- Many existing methods to calculate CTOD can be costly and complicated, or apply only to particular configurations

- A new numerical method for calculating CTOD was investigated
  - NASGRO's Boundary Element module NASBEM was adapted to calculate displacements at any point on the crack
  - Demonstrated for a number of crack configurations:
    - finite and infinite domains
    - center and edge cracks
    - complex cases with several cracks and holes
  - Great accuracy at minimal computational cost

Future

- Still a work in progress:
  - CTOA investigated … more work needs to be done
  - Is K_c corrected for Dugdale plastic zone size a better fracture criterion than $K_c(a)$ alone, or K_c corrected for Irwin plastic zone size?
  - Multi-site damage issues to be investigated
  - Strain hardening -- easy to model

- CTOD capability is currently still a research tool
  - Turn capability into production-level tool
  - Implement automation of CTOD calculations
    - ⇨ Manual meshing and convergence to $K=0$ for multiple cracks is tedious!